# Unit 3
## GAME THEORY

## Lesson 31

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**Learning Objective:**

- Analyze different types of non zero sum games

  - ➤ Hawk vs. Dove game.
  - ➤ Advertising problem.
  - ➤ Problem of companies polluting the environment.

- Rationalizability or Iterative Elimination of Dominated Strategies.

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Hello students,

**In the previous lectures you have learned the concept of solution for non-zero sum games.**

**So let us today apply it on a very important illustration……..**

# Non-zero sum games: example – Hawk vs. Dove

- Suppose you have two countries, A and B, which share a border. There is an offshore oilfield which both of the countries claim belongs to them.

- Suppose that the oil field has value = V .

- We can simplify the possible outcomes facing the countries into: (1) they decide to share the oil field 50-50 without fighting a war, (2) country A threatens war and country B surrenders, (3) country B threatens war and country A surrenders, or (4) they both fight a war.

- We can simplify the strategy choice for each country as (1) fight a war or be willing to fight a war (Hawk), or (2) not be willing to fight a war (Dove).

- If we make the additional assumption that the countries are evenly matched militarily, and that the total cost of fighting a war is = C we can model this situation as a non-zero sum game with the following payoff table:

|  |  | B | |
|---|---|---|---|
|  |  | Hawk | Dove |
| A | Hawk | $\frac{(V-C)}{2}$ , $\frac{(V-C)}{2}$ | $V$ , $0$ |
|  | Dove | $0$ , $V$ | $\frac{V}{2}$ , $\frac{V}{2}$ |

- The entry corresponding to the Hawk-Hawk combination is $(V - C) / 2$ for both countries.

- This assumes that if both countries play the Hawk strategy then they will fight a war with cost C and then they still end up splitting the resources after the war.

- This could be interpreted as a deterministic quantity if we were certain that the outcome of the war were a draw.

- However we can also consider the outcome of the war as a stochastic quantity and use the expected value of the outcome for the Hawk-Hawk strategy, this is more realistic since the outcome of the war would not be known with certainty.

|  |  | B | |
|---|---|---|---|
|  |  | Hawk | Dove |
| A | Hawk | $\dfrac{(V-C)}{2}$ , $\dfrac{(V-C)}{2}$ | V , 0 |
|  | Dove | 0 , V | $\dfrac{V}{2}$ , $\dfrac{V}{2}$ |

- If we still assume that the countries are evenly matched then we can consider the case in which before the war is fought
  Pr(A wins) = Pr(B wins) = 0:50 or
  Pr(A wins) = Pr(B wins) = Pr(draw) = 1

  What is the expected value for the Hawk-Hawk combination in this case?

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- How to solve this game?

- Non-zero sum games often involve a combination of cooperation and competition built into the outcomes.

- If we think in terms of the overall good for both countries what seems to be the best possible outcome? What is the worst possible outcome?

  **Can you identify?**

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|  |  | B | |
|---|---|---|---|
|  |  | Hawk | Dove |
| A | Hawk | $\dfrac{(V-C)}{2}$ , $\dfrac{(V-C)}{2}$ | V , 0 |
|  | Dove | 0 , V | $\dfrac{V}{2}$ , $\dfrac{V}{2}$ |

- If we want to find a dominant strategy equilibrium we suppose that player A reasons as follows: if player B plays Dove, what is my best strategy in terms of the best possible outcome? if player B plays hawk, what is my best strategy ? We also assume player B thinks similarly

- This outcome is a dominant strategy equilibrium, if either player unilaterally changes strategy then they will end up with a worse result.

- From the table, under what conditions, on V and C will there be a dominant strategy equilibrium, and what will be the outcome that corresponds to this equilibrium ?

- In the Hawk-Dove game, if V > C we have a result similar to the Prisoner's Dilemma game in which the sub optimal strategy, in this case Hawk-Hawk, in Prisoner's Dilemma.

  Confess-Confess is a dominant strategy equilibrium solution.


## Non zero sum game Another illustration:

Now, can you answer a simple question?

Why do firms spend so much money on advertising ?

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So let us implement this on an example:

- Consider two large competing companies each with a 0.5 market share and thus a 50% share of the total available profits P.

- The firms need to choose whether or not to spend money on advertising their products.

- Suppose that if neither of the firms advertises then they will maintain the current market share i.e. 50 - 50.

- However suppose that if one of the firms advertises and the other does not, the firm that advertises will get an increase in their proportion by I, thus the other firm will lose I.

- Suppose that the cost of launching an advertising campaign is equal to C. What will the payoff table look like for the two companies:

|  |  | B | |
|---|---|---|---|
|  |  | Advertise | Don't |
| A | Advertise |  |  |
|  | Don't |  |  |

- Under what conditions will there be a dominant strategy equilibrium(ia), and what will it(they) be ? Does this seem optimal ?

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## Illustration 3

Why do companies (or individuals) pollute the environment ?

- Suppose you have N companies who need to choose between using methods that are cheaper and cause considerable pollution, or methods that don't pollute but are more expensive, and thus lead to smaller profits.

- Suppose that the cost to each company that results if all companies choose to pollute is D, this is not a financial cost but the cost of having to breathe bad air, drink bad water etc.

- We can also assume that each individual company contributes D / N to the pollution.

- Suppose that the profit to each company that adopts the cheaper but polluting methods is P whereas the profit to companies that choose the more expensive non-polluting methods is P – C, thus C > 0 represents the additional cost of not polluting.

- To be realistic we would need to consider this as an N unit (company) game, but for simplicity we can consider one company, Company A, as the row player, making their decision, considering the possible moves of the other    N −1 companies, who then constitute the column player.

- If we assume that all of the other companies make the same decision, what does the payoff matrix look like:

|  |  | Other companies | |
|---|---|---|---|
|  |  | Pollute | Don't |
| A | Pollute |  |  |
|  | Don't |  |  |

For Company A, under what conditions will there be dominant strategy equilibrium (ia)? And what is(are) the equilibrium(ia) ?

**Returning to the Hawk vs. Dove game** what if V < C, i.e. the cost of fighting the war is more than the value of the resource. For example, suppose V = 4 and C = 6

|   |      | B        |        |
|---|------|----------|--------|
|   |      | Hawk     | Dove   |
| A | Hawk | -1 , -1  | 4 , 0  |
|   | Dove | 0 , 4    | 2 , 2  |

- In this case there will be no dominant strategy equilibrium.

- The best strategy for each participant depends on the strategy chosen by the other participant. You know that when there are no dominant strategies, we use equilibrium conception of Nash Equilibrium. We have a Nash Equilibrium if each participant chooses the best strategy, given the strategy chosen by the other participant.

|   |      | B        |        |
|---|------|----------|--------|
|   |      | Hawk     | Dove   |
| A | Hawk | -1 , -1  | 4 , 0  |
|   | Dove | 0 , 4    | 2 , 2  |

- In the example,
  if A opts for "Hawk", then B is better off choosing "Dove". Likewise if B chooses "Dove", A is better off choosing "Hawk". So the upper right hand corner corresponding to A playing "Hawk" and B playing "Dove" is a Nash-equilibrium.

- To check if a particular strategy pair is a Nash equilibrium see what happens if each player unilaterally switches strategies. If neither can be made better off by switching then the pair is a Nash equilibrium.

- For this example if A unilaterally switches from "Hawk" to "Dove" then the payoff drops from 4 to 2 so A is worse off switching, if B switches from "Dove" to "Hawk" B's payoff drops from 0 to −1.

- Every dominant strategy equilibrium is also a Nash equilibrium, however not every Nash equilibrium is a dominant strategy equilibrium and, in fact, a game can have more than one Nash equilibria.

- If we return to the previous example, in addition to the upper right hand corner, is there another Nash equilibrium ?

|  |  | B | |
|---|---|---|---|
|  |  | Hawk | Dove |
| A | Hawk | -1 , -1 | 4 , 0 |
|  | Dove | 0 , 4 | 2 , 2 |

- Note in this case the actual outcome of the game is uncertain.

## Other methods for "solving" games: Rationalizability or Iterative Elimination of Dominated Strategies

- Consider the following non-zero sum game matrix:

|  |  | B | | |
|---|---|---|---|---|
|  |  | 1 | 2 | 3 |
| A | 1 | (10; 10) | (12; 8) | (10; 5) |
|  | 2 | (6; 11) | (7; 12) | (15; 5) |
|  | 3 | (5; 5) | (5; 15) | (5; 20) |

- To solve this game we first assume perfect information, i.e. each player knows their own payoffs and the other player's payoffs for each set of

strategies. We also assume that each player acts rationally and that each player knows that the other player will act rationally.

- If we consider the game for player B's point of view we can see that no column is dominated by any other column. If A plays 1, B's best choice is 1, if A plays 2, B's best choice is 2, and if A plays 3 B's best choice is 3.

- However if B examines the game from A's point of view, B realizes that A has no reason to play strategy 3 which for A is dominated by both strategy 1 and 2.

- Therefore B needs only consider the reduced 2 by 3 matrix:

|   |   | B |   |   |
|---|---|---|---|---|
|   |   | 1 | 2 | 3 |
| A | 1 | (10; 10) | (12; 8) | (10; 5) |
|   | 2 | (6; 11) | (7; 12) | (15; 5) |

- For this matrix B now has a dominated strategy, strategy 3. B therefore eliminates strategy 3 from consideration.

- This leaves:

|   |   | B |   |
|---|---|---|---|
|   |   | 1 | 2 |
| A | 1 | (10; 10) | (12; 8) |
|   | 2 | (6; 11) | (7; 12) |

- B now has only to choose between 1 and 2, neither of which dominates the other. However B assumes that A knows that B has eliminated strategy 3 from consideration, so given this fact A now knows that the best strategy for A to play is 1 since in the reduced matrix, row 1 dominates row 2. Since B also knows this B knows A should play strategy 1, therefore B also plays strategy 1.

NOW

- Apply the rationalizability method to solve the following game matrix

|   |   | B | | |
|---|---|---|---|---|
|   |   | 1 | 2 | 3 |
| A | 1 | (7; 2) | (9; 5) | (7; 7) |
|   | 2 | (2; 17) | (2; 12) | (2; 2) |
|   | 3 | (12; 2) | (4; 9) | (3; 8) |

**So,** now let us summarize today's discussion:

Summary
We have discussed about:

- Hawk vs. Dove game.
- Advertising problem.
- Problem of companies polluting the environment.
- Rationalizability or Iterative Elimination of Dominated Strategies.